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MACHINE LEARNING AND ASSET MANAGEMENT

BENJAMIN BRUDER

ASSET MANAGEMENT BY LYXOR

AGENDA

PORTFOLIO ALLOCATION PRINCIPLES 3

TREND ESTIMATION PROBLEM 6

PITFALLS 10

RISK FACTORS 15

PORTFOLIO ALLOCATION PRINCIPLES

MODERN PORTFOLIO THEORY (1952)

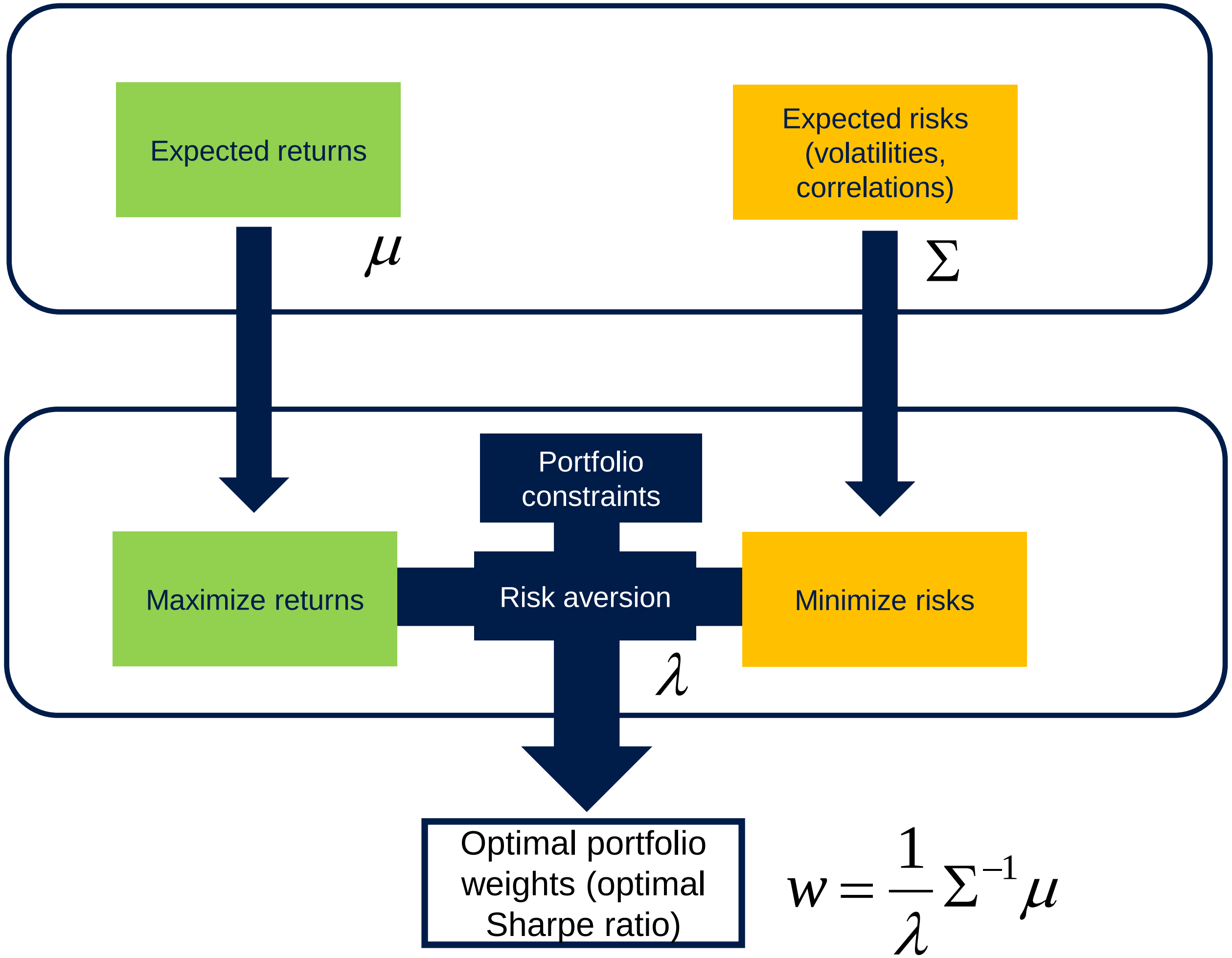
Expectations over investment universe

(i.e. probability distribution of asset prices)

Statistics

Portfolio optimization problem

Convex analysis



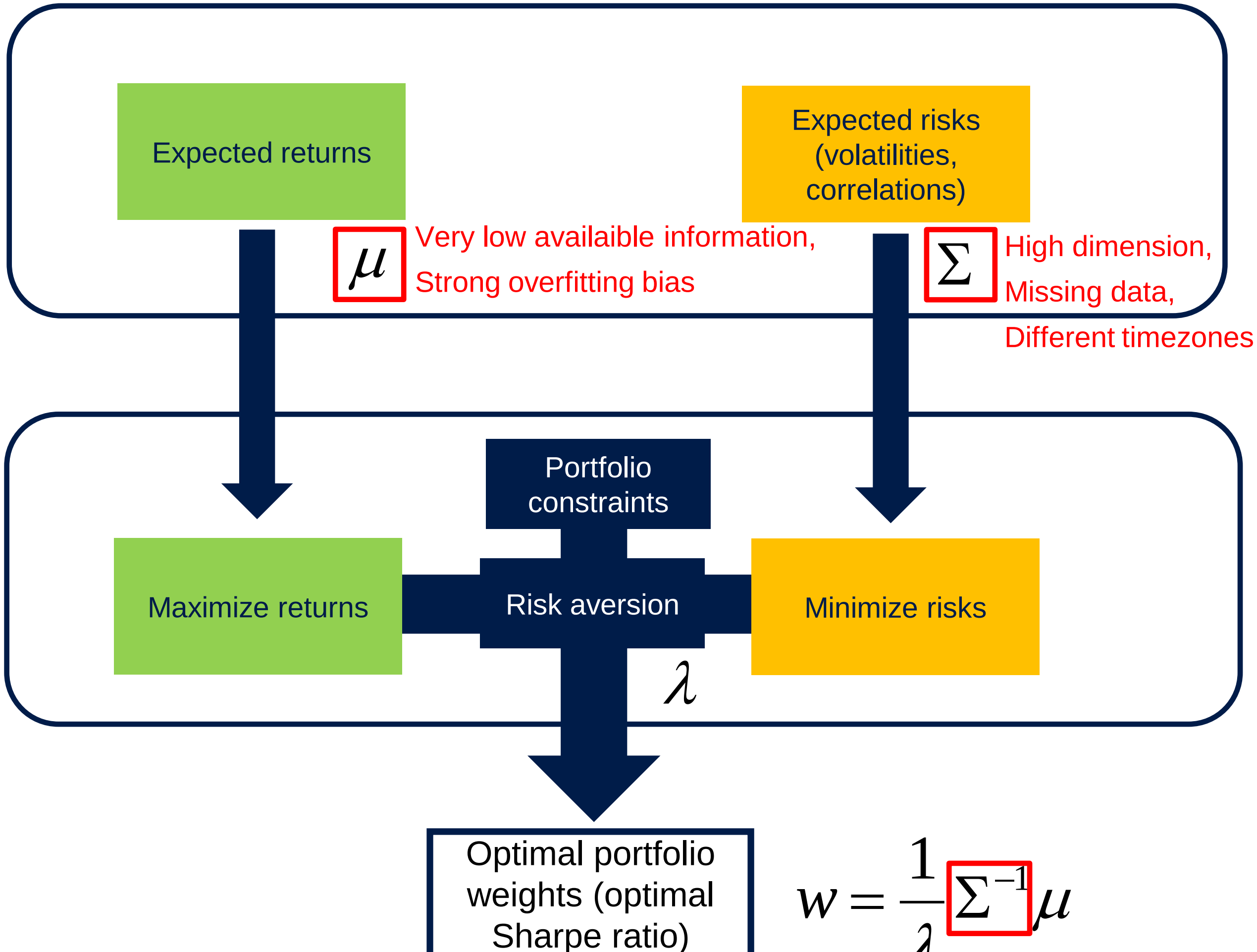
$$w = \frac{1}{\lambda}\Sigma^{-1}\mu$$

MODERN PORTFOLIO THEORY (1952): PROBLEM LINKED WITH WITH MODERN PRACTICES

Expectations over investment universe

(i.e. probability distribution of asset prices)

Optimization problem



$$w = \frac{1}{\lambda} \Sigma^{-1} \mu$$

Potential instability

Statistics

Convex analysis

TREND ESTIMATION PROBLEM

WHAT IF PARAMETERS ARE CONSTANT?

SIMPLEST POSSIBLE MODELLING

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

Average return

Best estimation: Average long term return

Estimation quality depends on price history total length

$$\mu = \frac{1}{T}\ln\left(\frac{S_T}{S_0}\right) + \frac{1}{2}\sigma^2 - \frac{\sigma}{T}W_T$$

Periode	Moyenne	Ecart Type	Proba>0	Bruit/Signal
1M	10%	54%	57%	541%
3M	10%	30%	63%	300%
6M	10%	21%	68%	212%
1Y	10%	15%	75%	150%
3Y	10%	9%	88%	87%
5Y	10%	7%	93%	67%
10Y	10%	5%	98%	47%

Risk

Best estimation: Quadratic variations

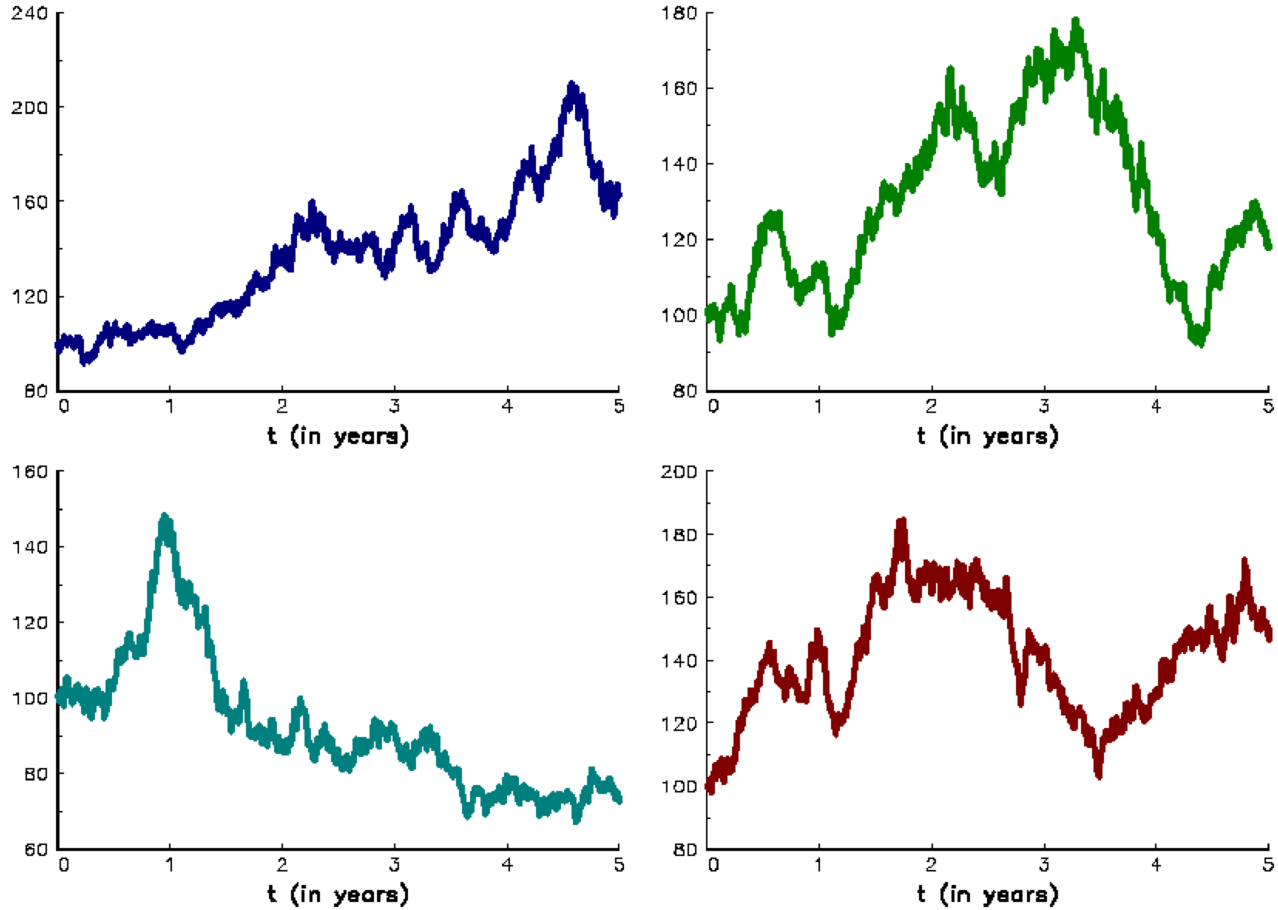
Estimation quality depends on overall numbers of returns

$$\hat{\sigma}^2 = \frac{1}{t_n - t_0}\sum_{i=1}^{n}\left(\frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}}\right)^2$$

periode	nb prix	Moyenne	Ecart type	Bruit/Signal
10D	10	14.6%	3.3%	23%
1M	20	14.8%	2.4%	16%
3M	60	14.9%	1.4%	9%
6M	130	15.0%	0.9%	6%
1Y	260	15.0%	0.7%	4%

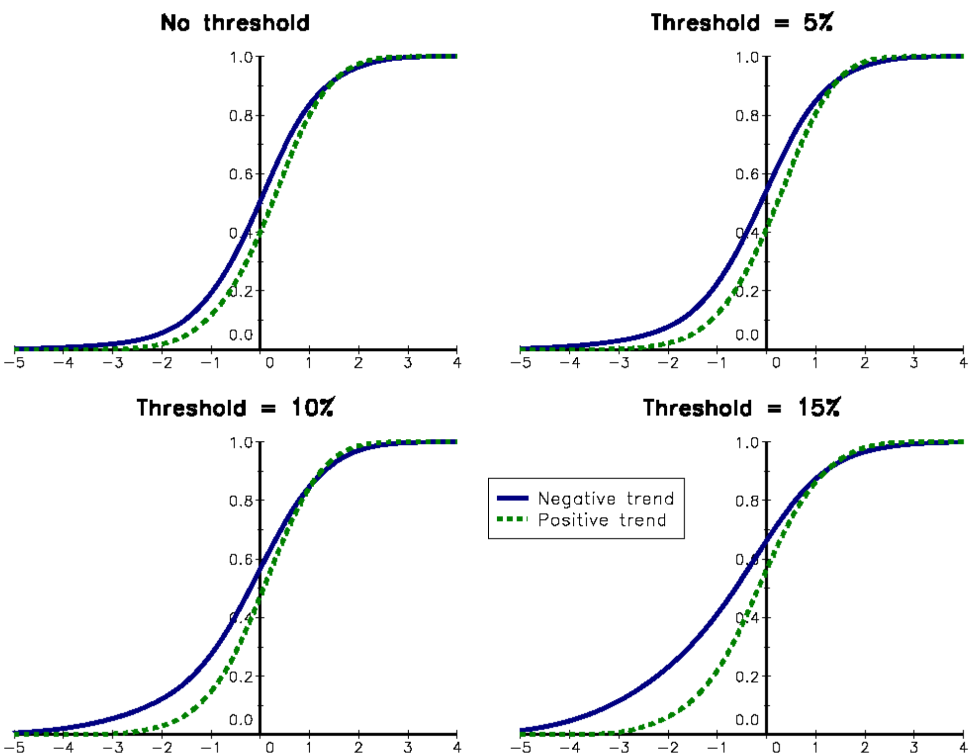
In practice, parameters vary with time (sure for volatility, we suppose so for trends…)

WHICH ARE THE UNDERLYING TRENDS OF THESE PRICES?



HISTORICAL VERIFICATION OF TREND PERSISTENCE

- Distribution of 1M GSCI returns conditionally the past 3M return



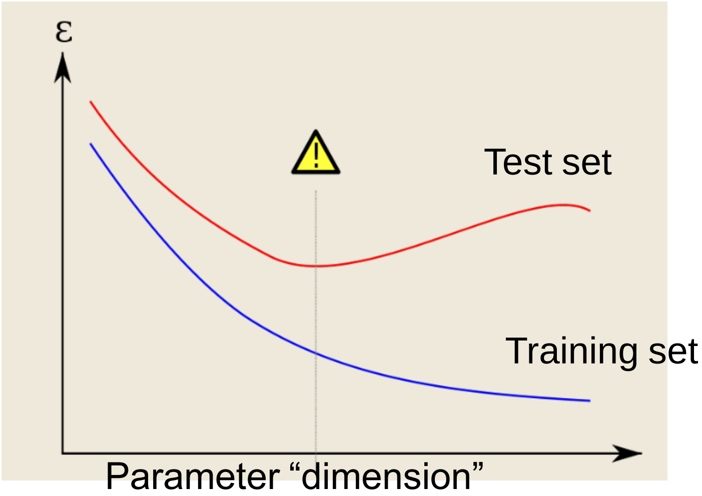
Trend	Positive	Negative	Difference
Eurostoxx 50	1.1%	0.2%	0.9%
S&P 500	0.9%	0.5%	0.4%
MSCI WORLD	0.6%	−0.3%	1.0%
MSCI EM	1.9%	−0.3%	2.2%
TOPIX	0.4%	−0.4%	0.9%
EUR/USD	0.2%	−0.2%	0.4%
USD/JPY	0.2%	−0.2%	0.4%
GSCI	1.3%	−0.4%	1.6%

PITFALL: OVERFITTING WITH FEW INFORMATION

ARE STATISTICAL LEARNING CONCEPTS INSIGHTFUL OR MISLEADING IN ASSET MANAGEMENT?

Underfitting vs overfitting (aka bias vs variance)

Typical pattern on a machine learning problem:



Dimension: if we consider thousands of parameters, there has to be some strategies that performed in the past

PORTFOLIO OPTIMIZATION WITH STANDARD ESTIMATORS

OPTIMIZATION WITHOUT CONSTRAINTS

Initial asset universe n asset with (estimated) covariance matrix Σ

Perform a PCA, obtain independent portfolios, with unit variance. The new covariance matrix is the identity matrix.

Equivalent problem: Allocation on those portfolios which are (drifted) Brownian motions $\left(W^1, ..., W^n\right)$

Standard estimation of the drift on the interval $[0, T]$: $\hat{\mu} = \frac{1}{T}\left(W_T^1, ..., W_T^n\right)$

Perform Markowitz optimization problem, i.e. maximize: $\alpha'\hat{\mu} - \frac{1}{2}\lambda\alpha' I\alpha$

Optimal portfolio compsition is given by: $\alpha^* = \frac{1}{\lambda}\hat{\mu} = \frac{1}{\lambda T}W_T$

Sharpe ratio of the optimal portfolio: $\frac{\alpha^*.\hat{\mu}}{\sqrt{(\alpha^*)' I\alpha^*}} = \frac{\sqrt{\sum_{i=1}^{n}\left(W_T^i\right)^2}}{T}$

WHAT CAN WE "LEARN" FROM A WHITE NOISE?

SUPPOSE THAT ALL OUR ASSETS ARE ZERO-MEAN BROWNIAN MOTIONS

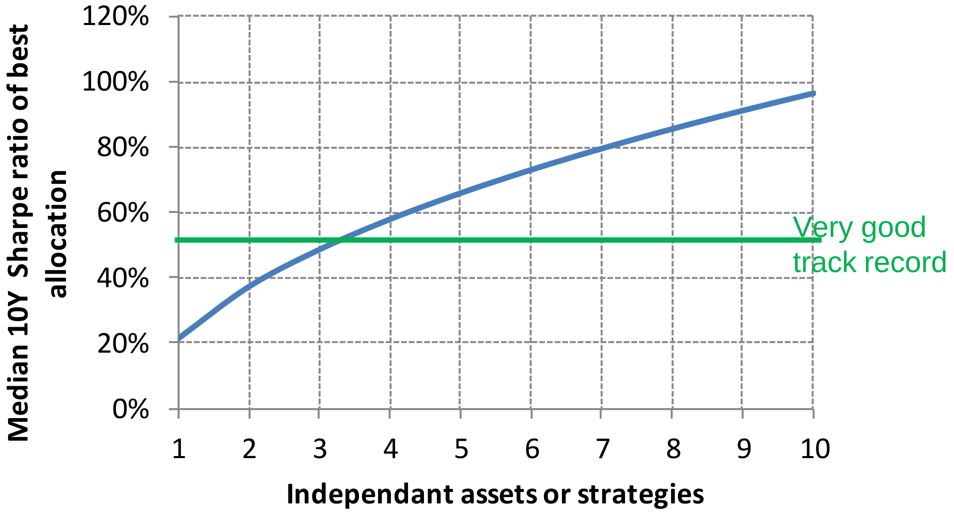
In sample bias:

Maximize the ex post Sharpe ratio of a combination of n Brownian Motions.

Best ex-post portfolio allocation: $\left(W_T^1, \ldots, W_T^n\right)$

Best Sharpe ratio:

$$\frac{\sqrt{\sum_{i=1}^{n}\left(W_T^i\right)^2}}{T} \sim \frac{\sqrt{\chi^2(n)}}{\sqrt{T}}$$



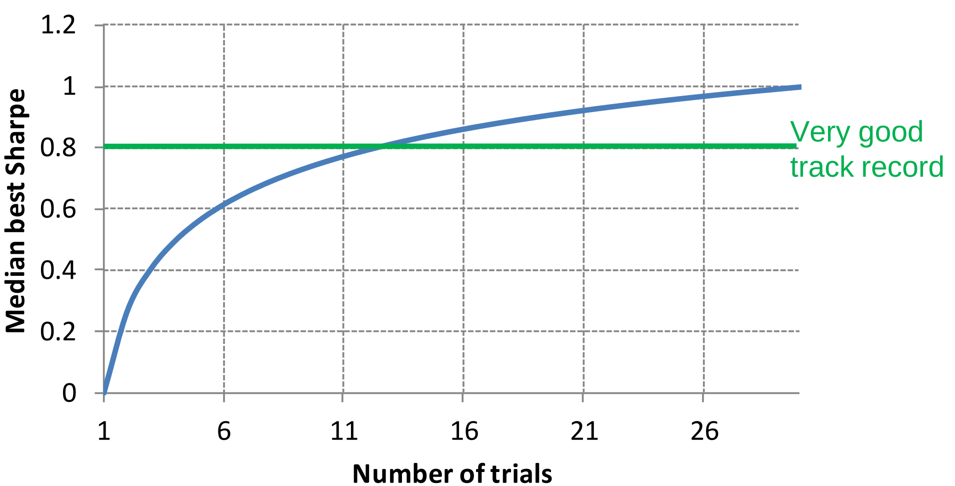
Out of sample bias:

Try n (Brownian) strategies.

Keep the best out of sample performer on a given test set of 4 years.

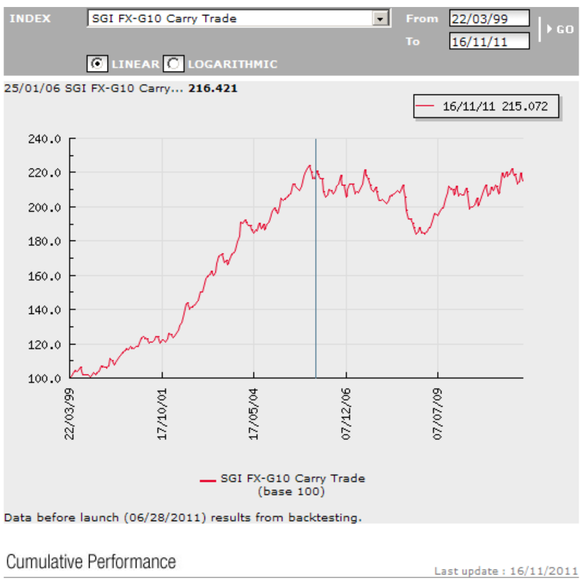
Best Sharpe ratio:

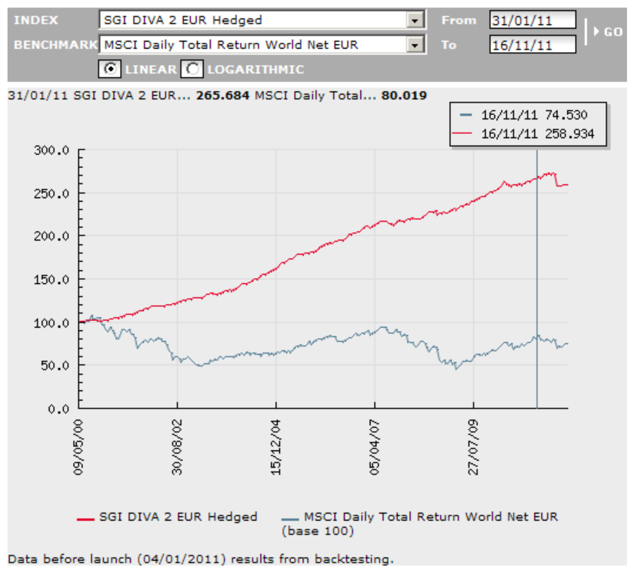
$$\max_i (X_i) \text{ where } X_i \sim \mathcal{N}\left(0, \frac{1}{\sqrt{T}}\right)$$

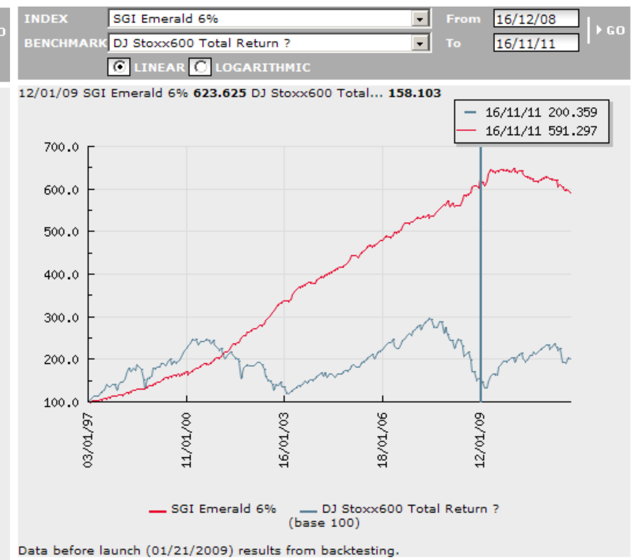


EXAMPLE: INVESTMENT BANK INDICES

A large number of « reasonable » investment strategies that performed in the past







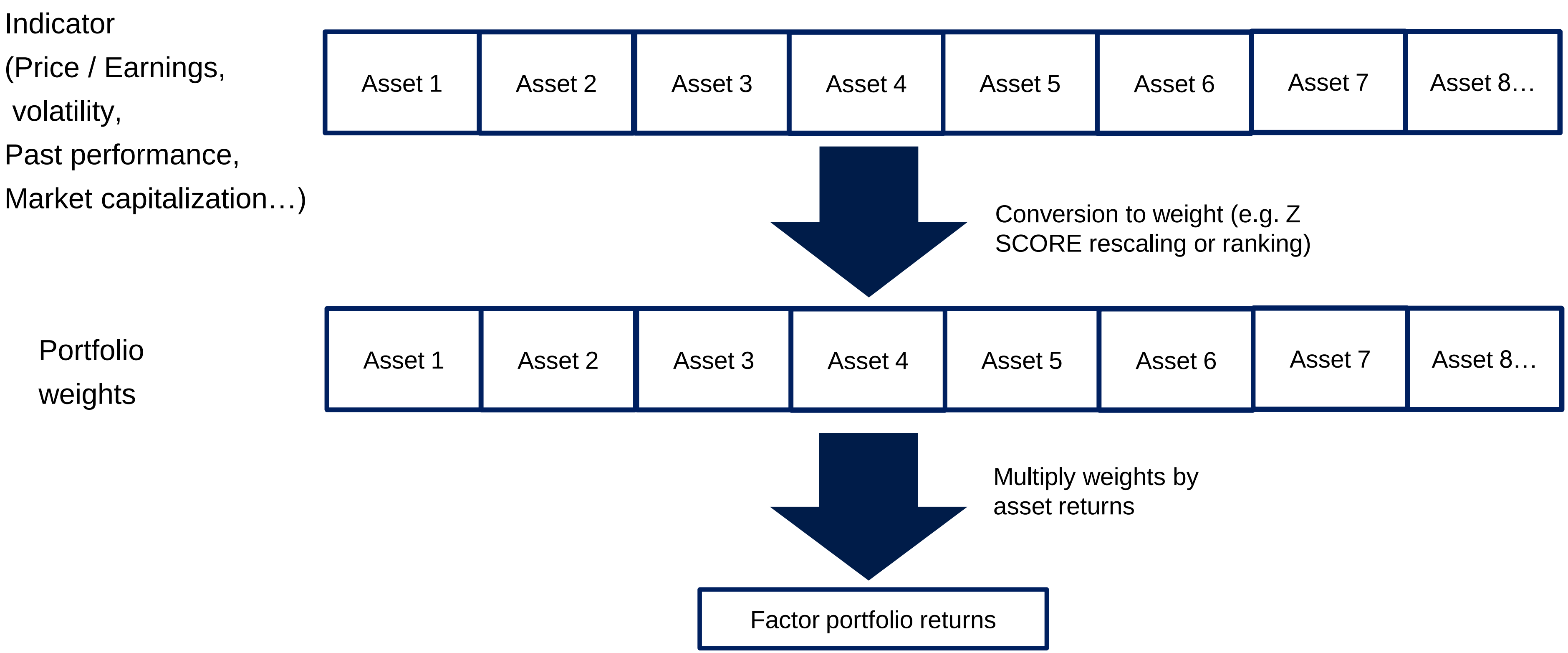
POSITIVE RESULT: RISK FACTORS

FACTOR INVESTING: HOW TO BUILD FACTORS

HOW TO COMPENSATE SHORT HISTORY WITH MULTIPLICITY OF ASSETS

Can we find variables (price/earning, past returns, volatility…) explaining the covariance structure?

•Built a portfolio weighted from those variables (rescaled…)



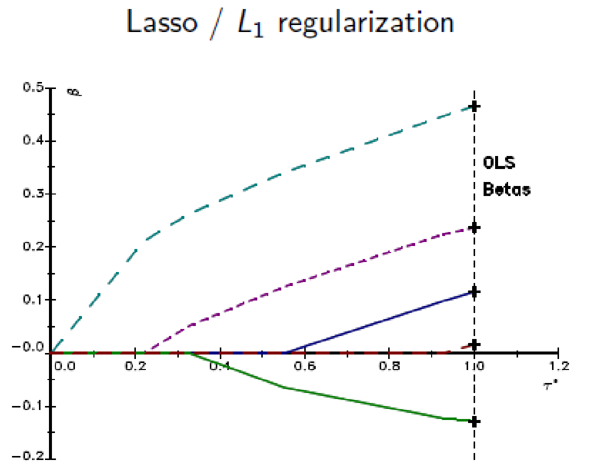
FACTOR INVESTING

ANALYZE

•See if the factor portfolios explain the returns (regression of given portfolios vs factor portfolio returns)

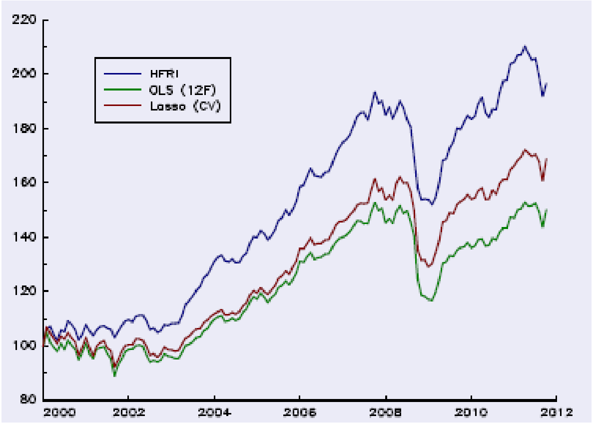
$$R_t^{GivenPortfolio} = \beta_{factor1} R_t^{factor1} + \beta_{factor2} R_t^{factor2} + \ldots + \beta_{factorn} R_t^{factorn} + \varepsilon_t$$

•Factor Selection : LASSO regression



•Dynamic factor Exposure: Ordinary least square on rolling window or Kalman filtering

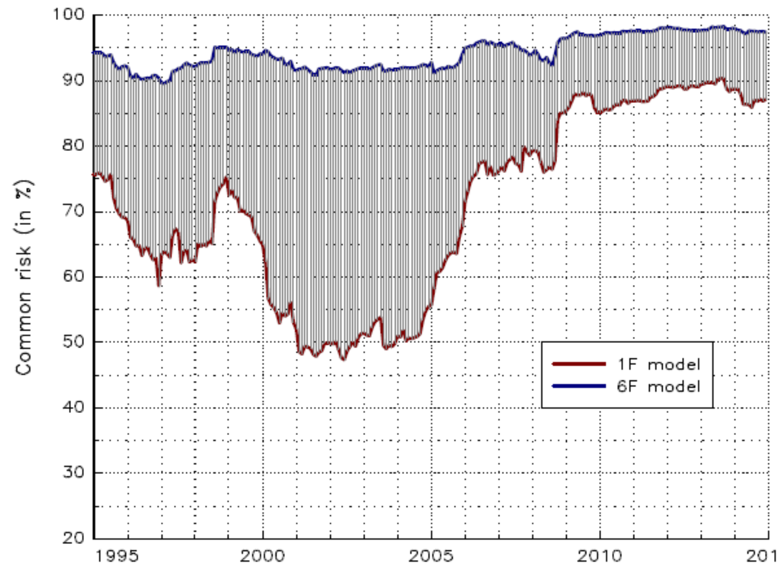
Example: Hedge fund replication



Defining the right risk factors

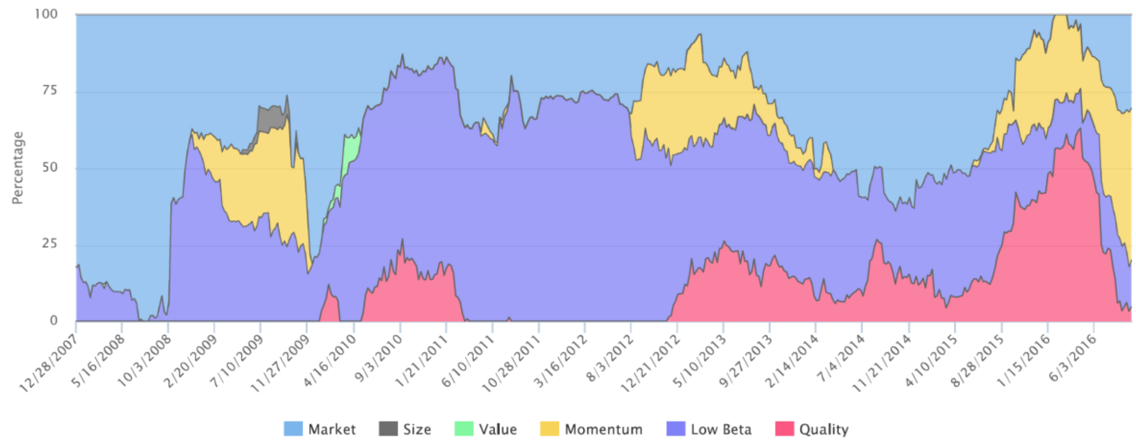
» Decomposition of equity portfolio returns

- **Market risk factor**
- **Size factor**
- **Value factor**
- **Momentum factor**
- **Low Risk factor**
- **Quality factor**



» Large proportion of stock returns explained by those factors

» Example: Fund analysis

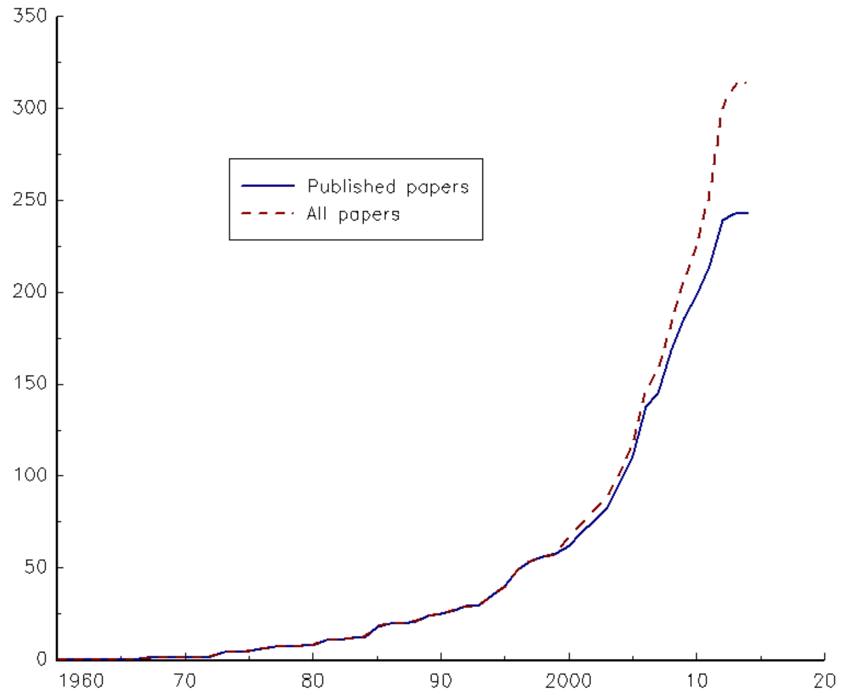


BUT….

STILL, NEED TO KEEP IN MIND OUR NATURAL BIAS

"*Standard predictive regressions fail to reject the hypothesis that the party of the **U.S. President**, the **weather in Manhattan**, **global warming**, **El Nino**, **sunspots**, or the **conjunctions of the planets**, are significantly related to anomaly performance. These results are striking, and quite surprising. In fact, some readers may be inclined to reject some of this paper's conclusions solely on the grounds of plausibility. I urge readers to consider this option carefully, however, as doing do so entails rejecting the standard methodology on which the return predictability literature is built.*"
(Novy-Marx, 2014, Journal of Financial Economics)

Cumulative number of risk factors



CONCLUSION

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DATA SCIENCE IS APPEALING, BUT INFORMATION MAY BE TOO SMALL COMPARED TO NOISE

•The general mindset of machine learning (training/validation/test) gives good insights.

•Techniques apply well when studying covariances (time series or cross sectional).

•Need to be parsimonious, especially when estimating expected returns.

	Bond Scoring	Stock Picking	Trend Filtering	Mean Reverting	Index Tracking	HF Tracking	Stock Class.	Technical Analysis
Lasso		☺	☺	☺	☹	☺		
NMF							☺	☹
Boosting		☺				☺		
Bagging		☺				☺		
Random forests	☺			☹				☹
Neural nets	☺					☹		
SVM	☺	☹	☹				☹	
Sparse Kalman					☹	☺		
K-NN	☹							
K-means	☺						☺	
Testing protocols[2]	☺	☺	☺	☺		☺		

☺ = encouraging results

☹ = disappointed results