Hyperbolic discounting (A case of time inconsistency)

- Interest rates are higher for longer time horizons.
- Quasi-hyperbolic discounting structure (Laibson 1997): Time is divided into two periods. The present period and all the future periods.
- Payoffs in the current period are discounted by a discounted rate $\gamma$.
- Payoffs in a future period which is $\tau$-period ($\tau > 1$) away from the current period is discounted by $\gamma^\tau$ and then further discounted by an additional factor $\lambda \in (0,1)$. 

- Interest
- Quasi-linear divided into two periods
- Payoffs
- Payoffs in the current period
Optimal Dividend Strategy with Time-Inconsistent Preferences and Cost Constraints

Zhonghe Li
Joint work with Shutong Chen and Yan Zeng

Sun Yat-sen Business School, Sun Yat-sen University
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- State space:
  \[ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \]

- A set of admissible control:
  \[ \mathcal{A} = \{ u \in \mathbb{R}^2 : u_t = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \} \]

- Value function in a CTS:
  \[ V(x) = \min_{u \in \mathcal{A}} \left\{ \int_0^\infty e^{-\rho t} \left( \frac{1}{2} x_1^2 + u_1^2 \right) dt + 0 \right\} \]

- Subject to state equation (1)-(3):
  \[ \frac{dx}{dt} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad x(0) = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \]

- Stopping time.
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